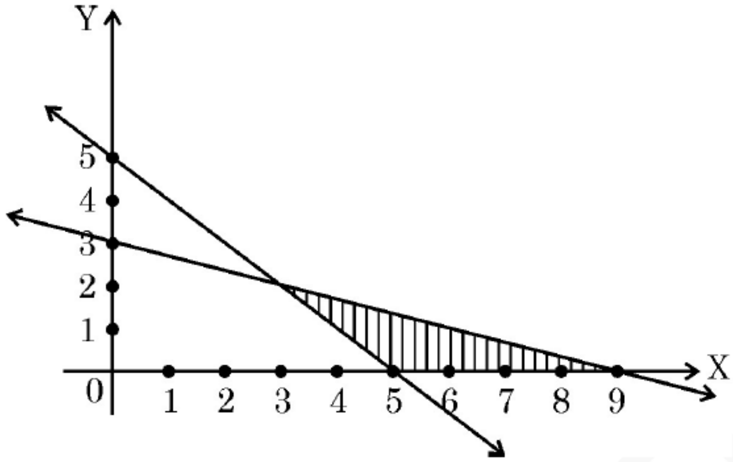
MARKING SCHEME Strictly Confidential (For Internal and Restricted use only) Senior Secondary School Examination, 2026 MATHEMATICS (041) (PAPER CODE 65/2/3)	
General Instructions: -	
1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	**“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS.”**
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4.	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9.	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/2/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A **Q. Number 1 to 20 are multiple choice questions of 1 mark each.**		
1.	Direction cosines of line $\frac{1-x}{0} = y = z$ are (A) 1, 1, 1 (B) $0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$ (C) 1, 0, 0 (D) 0, –1, –1		
Sol.	(B) $0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$		1
2.	In a linear programming problem, the linear function which has to be maximized or minimized is called (A) a feasible function (B) an objective function (C) an optimal function (D) a constraint		
Sol.	(B) an objective function		1
3.	For the feasible region shown below, the non-trivial constraints of the linear programming problem are  (A) $x + y \le 5, x + 3y \le 9$ (B) $x + y \le 5, x + 3y \ge 9$ (C) $x + y \ge 5, x + 3y \le 9$ (D) $x + y \ge 5, 3x + y \le 9$		
Sol.	(C) $x + y \ge 5, x + 3y \le 9$		1

4.	For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$ (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$ (C) $\dfrac{1 - P(A \cap B)}{P(B')}$ (D) $\dfrac{1 - P(A \cup B)}{P(B')}$	
Sol.	(D) $\dfrac{1 - P(A \cup B)}{P(B')}$	1
5.	A relation R on set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 3), (3, 3), (1, 1), (2, 2), (3, 1)\}$ is (A) only reflexive and symmetric (B) reflexive only (C) only reflexive and transitive (D) reflexive, symmetric and transitive	
Sol.	(D) reflexive, symmetric and transitive	1
6.	If A and B are square matrices of same order, then which of the following statements is/are always true ? (i) $(A + B)(A - B) = A^2 - B^2$ (ii) $AB = BA$ (iii) $(A + B)^2 = A^2 + AB + BA + B^2$ (iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$ (A) Only (i) and (iii) (B) Only (ii) and (iii) (C) Only (iii) (D) Only (iii) and (iv)	
Sol.	(C) Only (iii)	1
7.	If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is (A) 2 (B) 6 (C) 4 (D) 0	
Sol.	(A) 2	1

8.	If $A = \begin{bmatrix} \tan x & \cot x \\ -\cot x & \tan x \end{bmatrix}$ and $A + A' = 2I$, then value of $x \in \left[0, \frac{\pi}{2}\right]$ is (A) 0 (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$	
Sol.	(C) $\frac{\pi}{4}$	1
9.	For a square matrix A, $(3A)^{-1} =$ (A) $3A^{-1}$ (B) $9A^{-1}$ (C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$	
Sol.	(C) $\frac{1}{3}A^{-1}$	1
10.	If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is (A) 4 (B) 5 (C) –4 (D) 9	
Sol.	(C) –4	1
11.	If $x + y = xy$, then $\frac{dy}{dx}$ is (A) $\frac{y}{x-1}$ (B) $\frac{1}{x-1}$ (C) $\frac{y-1}{x-1}$ (D) $\frac{1-y}{x-1}$	
Sol.	(D) $\frac{1-y}{x-1}$	1
12.	$\int \frac{dx}{\sec x + \tan x}$ is equal to (A) $\log\lvert \sec x + \tan x \rvert + C$ (B) $\log\lvert \sec x - \tan x \rvert + C$ (C) $\log\lvert 1 + \cos x \rvert + C$ (D) $\log\lvert 1 + \sin x \rvert + C$	
Sol.	(D) $\log\lvert 1 + \sin x \rvert + C$	1

13.	For $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ (A) local maximum value is 2 (B) local minimum value is –2 (C) local maximum value is –2 (D) local minimum value < local maximum value	
Sol.	(C) local maximum value is –2	1
14.	Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$? (A) $\int_0^4 x^2 \, dx$ (B) $2\int_0^4 x^2 \, dx$ (C) $\int_0^{16} \sqrt{y} \, dy$ (D) $2\int_0^{16} \sqrt{y} \, dy$	
Sol.	(D) $2\int_0^{16} \sqrt{y} \, dy$	1
15.	The general solution of the differential equation : $x^2 dy + y^2 dx = 0$ is (A) $x^3 + y^3 = k$ (B) $\frac{1}{y} - \frac{1}{x} = k$ (C) $\frac{1}{y} + \frac{1}{x} = k$ (D) $\log y^2 + \log x^2 = k$	
Sol.	(C) $\frac{1}{y} + \frac{1}{x} = k$	1
16.	The integrating factor of the differential equation $2x\frac{dy}{dx} - y = 3$ is (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$ (C) e^x (D) e^{-x}	
Sol.	(B) $\frac{1}{\sqrt{x}}$	1

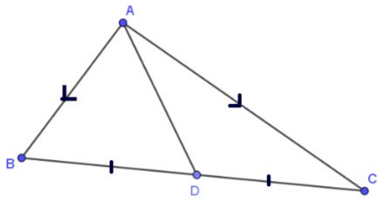
17.	If $\lvert \vec{a} \rvert = 5$ and $-2 \le \lambda \le 1$, then the sum of greatest and the smallest value of $\lvert \lambda \vec{a} \rvert$ is (A) -5 (B) 5 (C) 10 (D) 15	
Sol.	(C) 10	1
18.	Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is (A) $\hat{i} + 2\sqrt{2}\,\hat{j}$ (B) $3\hat{k}$ (C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$ (D) $\sqrt{3}\,\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{3}\,\hat{k}$	
Sol.	(C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$	1
	Assertion – Reason Based Questions **Direction :** Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	For two vectors $\vec{a}$ and $\vec{b}$ **Assertion (A) :** $\lvert \vec{a} \times \vec{b} \rvert^2 + (\vec{a} \cdot \vec{b})^2 = \lvert \vec{a} \rvert^2 \lvert \vec{b} \rvert^2$ **Reason (R) :** $\lvert \vec{a} \times \vec{b} \rvert = (\vec{a} \cdot \vec{b}) \tan\theta, \left(\theta \ne \frac{\pi}{2}\right)$	
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).	1

20.	**Assertion (A)** : A line can have direction cosines < 1, 1, 1 > **Reason (R)** : $\cos\theta = 1$ is possible for $\theta = 0$.		
Sol.	(D) Assertion (A) is false, but Reason (R) is true.		1

SECTION B **Q. Numbers 21 to 25 are very short answer questions of 2 marks each.**

21.	Find the co-ordinates of the point on line $x = \frac{y-1}{2} = \frac{z-2}{3}$ whose y co-ordinate is 3 times the x co-ordinate.		
Sol.	Let $x=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$ The coordinates of any point on this line is $A(\lambda, 2\lambda+1, 3\lambda+2)$ According to question, $2\lambda+1=3\lambda \Rightarrow \lambda=1$ $\therefore$ The required point is $A(1,3,5)$.	I II III	1 ½ ½
22.	(a) Check whether $f : R - \{3\} \rightarrow R$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not. **OR** (b) Check whether $f : Z \times Z \rightarrow Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.		
Sol. 22(a)	Let $y=\frac{x-2}{x-3}$ *i.e.* $x=\frac{3y-2}{y-1}$ Here $y \neq 1 \therefore R_f = R-\{1\} \neq$ Codomain $\therefore f$ is not onto.	I II	1 1
	OR		
Sol. 22(b)	Let $(x_1,y_1),(x_2,y_2) \in Z\times Z$ such that $f(x_1,y_1)=f(x_2,y_2)$ $\Rightarrow (2y_1,3x_1)=(2y_2,3x_2)$ $\Rightarrow y_1=y_2$, $x_1=x_2$ So, $(x_1,y_1)=(x_2,y_2)$ $\therefore f$ is injective.	I II	1 1

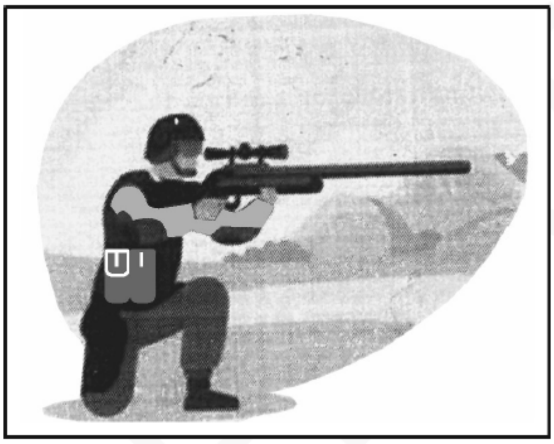
23.	If $x = e^{\sin^{-1} t}$, $y = e^{\cos^{-1} t}$, find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$		
Sol.	$x = e^{\sin^{-1} t} \Rightarrow \frac{dx}{dt} = e^{\sin^{-1} t} \times \frac{1}{\sqrt{1-t^2}}$ $y = e^{\cos^{-1} t} \Rightarrow \frac{dy}{dt} = e^{\cos^{-1} t} \times \frac{-1}{\sqrt{1-t^2}}$	I	1
	Now, $\frac{dy}{dx} = -\frac{e^{\cos^{-1} t}}{e^{\sin^{-1} t}}$ or $-e^{\cos^{-1} t - \sin^{-1} t}$	II	½
	$\left.\frac{dy}{dx}\right]_{t=\frac{1}{\sqrt{2}}} = -1$	III	½
	Note: Proportionate marks may be allotted if a student uses property $\sin^{-1} t + \cos^{-1} t = \frac{\pi}{2}$ to solve this question.		
24.	(a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$ **OR** (b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.		
Sol. 24(a)	$f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$ $f'(x) = -\sin x + 2\sin x \cos x$	I	1
	$f'(x) = 0 \Rightarrow \sin x(2\cos x - 1) = 0$ $\Rightarrow \sin x = 0$ or $\cos x = \frac{1}{2}$ $\therefore x = \frac{\pi}{3}$	II	½
	Now, $f(0) = 1$, $f\left(\frac{\pi}{3}\right) = \frac{5}{4}$, $f(\pi) = -1$ $\therefore$ Absolute maximum value of $f(x)$ is $\frac{5}{4}$.	III	½
	OR		

Sol. 24(b)	$V=\frac{2}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=2\pi r^2\frac{dr}{dt}=C$ (say) $\Rightarrow \frac{dr}{dt}=\frac{C}{2\pi r^2}$ Now, $S=3\pi r^2$ $\Rightarrow \frac{dS}{dt}=6\pi r\frac{dr}{dt}=6\pi r\times\frac{C}{2\pi r^2}=\frac{3C}{r}$ $\therefore S$ varies inversely as its radius.	I II	1½ ½
25.	If $\overrightarrow{AB}=\hat{j}+\hat{k}$ and $\overrightarrow{AC}=3\hat{i}-\hat{j}+4\hat{k}$ represent the two vectors along the sides AB and AC of ΔABC, prove that the median $\overrightarrow{AD}=\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$, where D is midpoint of BC. Hence, find the length of median AD.		
Sol.	Considering A as the origin, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are p.v. of B and C respectively. Hence p.v. of the mid point D is $\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$  So, $\overrightarrow{AD}=\frac{(\hat{j}+\hat{k})+(3\hat{i}-\hat{j}+4\hat{k})}{2}$ $\overrightarrow{AD}=\frac{3\hat{i}+5\hat{k}}{2}$ $\lvert\overrightarrow{AD}\rvert=\frac{1}{2}\sqrt{3^2+5^2}=\frac{\sqrt{34}}{2}$	I II III	1 ½ ½

SECTION C

Q. Numbers 26 to 31 are short answer questions of 3 marks each.

26. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.



The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit

(ii) atleast one shot misses the target.

OR

(b) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find P(E/F).

Sol. 26(a)	(i) Let probability of *not* hitting $= p$		
	$\therefore$ Probability of hitting $= 3p$		
	$p + 3p = 1 \Rightarrow p = \frac{1}{4}$		
	So, Probability of hitting, $P(H) = \frac{3}{4}$, Probability of *not* hitting, $P(\overline{H}) = \frac{1}{4}$	I	1
	Now, $P(\text{Target is hit}) = 1 - P(\overline{H}\,\overline{H}) = 1 - \frac{1}{4} \times \frac{1}{4} = \frac{15}{16}$	II	1
	(ii) $P(\text{at least one shot misses the target})$		
	$= 1 - P(HH) = 1 - \frac{3}{4} \times \frac{3}{4} = \frac{7}{16}$	III	1

OR

Sol. 26(b)	There are 3 people - Mother (M), Father (F), Son (S).		
	Sample Space $= \{MFS, MSF, FMS, FSM, SMF, SFM\}$		
	Total possible arrangements $= 6$	I	1
	Given E: Son on one end, F: Father in the middle		
	$P(E \cap F) = P(\text{Son on one end and Father in the middle}) = \frac{2}{6}$, $P(F) = \frac{2}{6}$	II	1
	$\therefore P(E \mid F) = \frac{P(E \cap F)}{P(F)} = 1$	III	1

	Alternative Method : For $P(E\|F)$, as F has already occured which means that the father is in the middle. So, son has to be on one end. $\therefore P(E\|F)=1 \quad$ (sure event)	I II	1 2
27.	Find : $\int \frac{3x-1}{\sqrt{x^2-4x}}\ dx$		
Sol.	$I=\int \frac{3x-1}{\sqrt{x^2-4x}}dx$ Put $3x-1=A(2x-4)+B$ $\Rightarrow A=\frac{3}{2}, B=5$ $\therefore I=\frac{3}{2}\int \frac{2x-4}{\sqrt{x^2-4x}}dx+5\int \frac{1}{\sqrt{x^2-4x}}dx$ $=3\sqrt{x^2-4x}+5\int \frac{1}{\sqrt{(x-2)^2-2^2}}dx$ $=3\sqrt{x^2-4x}\ +\ 5\log\left\|(x-2)+\sqrt{x^2-4x}\right\|+C$	I II III	1 1 1
28.	(a) Evaluate : $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$ **OR** (b) Evaluate : $\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} (\sin\ \|x\| + \cos\ \|x\|)\ dx$		

Sol. 28(a)	$I=\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}} \quad = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad (\text{i})$ Applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get $I=\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad (\text{ii})$ Adding (i) and (ii) $2I=\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1.dx$ $=[x]_{\pi/12}^{5\pi/12}=\frac{\pi}{3}$ $\Rightarrow I=\frac{\pi}{6}$	I II III IV	½ 1 1 ½
OR			
Sol. 28(b)	$I=\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} (\sin\|x\|+\cos\|x\|)dx$ $=\int_{\frac{-\pi}{6}}^{0}(-\sin x+\cos x)dx+\int_0^{\frac{\pi}{2}}(\sin x+\cos x)dx$ $=[\cos x+\sin x]_{-\pi/6}^{0}+[-\cos x+\sin x]_0^{\pi/2}$ $=\left(1-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)+(1+1)$ $=\frac{1}{2}(7-\sqrt{3})$	I II III	1 1 1
29.	If $\frac{d}{dx}(F(x))=\frac{1}{e^x+1}$, then find $F(x)$ given that $F(0)=\log\frac{1}{2}$.		

Sol.	$\frac{d}{dx}(F(x))=\frac{1}{e^x+1}$ $\Rightarrow F(x)=\int\frac{1}{e^x+1}dx$ $=\int\frac{e^{-x}}{1+e^{-x}}dx$ $=-\log(1+e^{-x})+C$ Now, $F(0)=-\log 2+C \Rightarrow \log\frac{1}{2}+\log 2=C \Rightarrow C=0$ $\therefore F(x)=-\log(1+e^{-x})$ Note: The above expression is equivalent to $F(x)=x-\log(e^x+1)$	I II III IV	1 1 ½ ½
30.	(a) Solve the following differential equation : $x\frac{dy}{dx}=y-x\sin^2\left(\frac{y}{x}\right)$, given that $y(1)=\frac{\pi}{6}$ **OR** (b) Find the general solution of the differential equation : $y\log y\frac{dx}{dy}+x=\frac{2}{y}$.		
Sol. 30(a)	Here, $\frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$ Put $\frac{y}{x}=v \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ Differential equation reduces to $v+x\frac{dv}{dx}=v-\sin^2 v$ i.e. $x\frac{dv}{dx}=-\sin^2 v$ $\Rightarrow -\int \operatorname{cosec}^2 v\,dv=\int\frac{dx}{x}$ $\Rightarrow \cot v=\log\|x\|+C$ $\Rightarrow \cot\left(\frac{y}{x}\right)=\log\|x\|+C$ Now $y(1)=\frac{\pi}{6}$ gives $C=\sqrt{3}$ Required solution is : $\cot\left(\frac{y}{x}\right)=\log\|x\|+\sqrt{3}$	I II III IV V	½ ½ ½ 1 ½
	OR		

Sol. 30(b)

Given differential equation can be written as

$$\frac{dx}{dy}+\frac{1}{y\log y}\cdot x=\frac{2}{y^2\log y}$$ I — 1

$$\text{I.F.}=e^{\int\frac{1}{y\log y}dy}=e^{\log(\log y)}=\log y$$ II — 1

Solution is given by

$$x\times\log y=\int\frac{2}{y^2\log y}\times\log y\,dy=\int\frac{2}{y^2}dy$$ III — ½

$$\Rightarrow x\log y=-\frac{2}{y}+C \quad\text{or}\quad xy\log y=-2+Cy$$ IV — ½

31. Solve the following linear programming problem graphically :

Maximize Z = $4500x + 5000y$

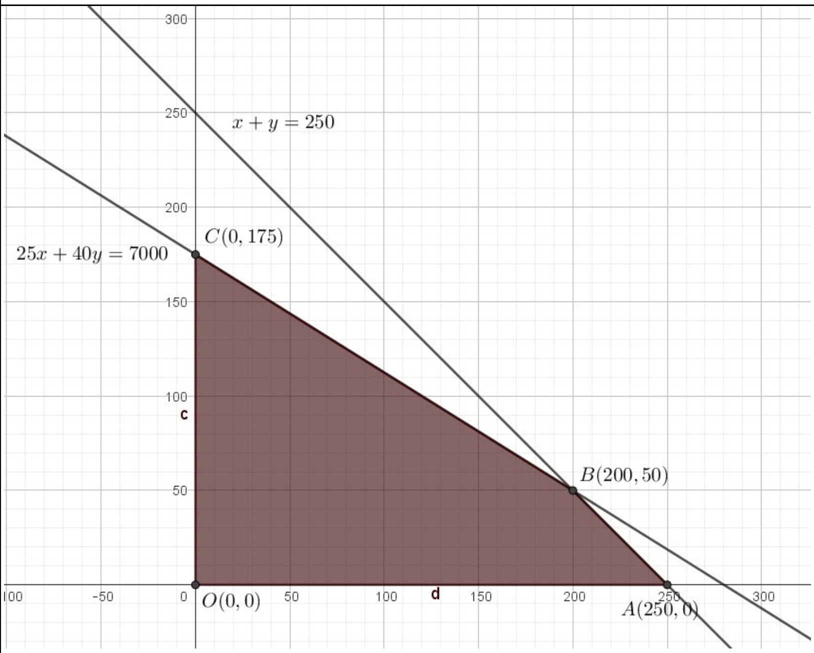
Subject to constraints

$x + y \leq 250$

$25x + 40y \leq 7000$

$x \geq 0,\ y \geq 0$

Sol.



I — For correct graph and shading 1½

Corner Point	Value of $Z=4500x+5000y$
$O(0,0)$	0
$A(250,0)$	1125000
$B(200,50)$	1150000
$C(0,175)$	875000

II — For correct table 1

$Z_{\max}=1150000$ when $x=200$ and $y=50$ III — ½

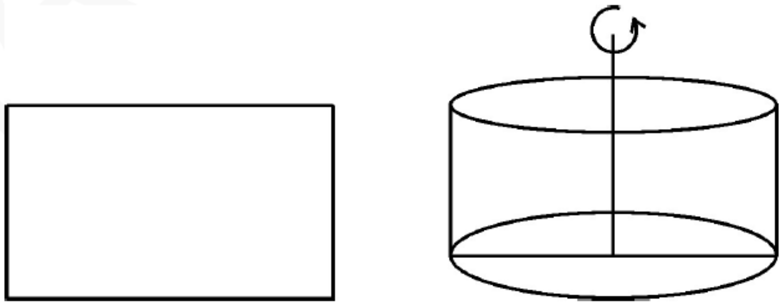
SECTION D

Q. Numbers 32 to 35 are long answer questions of 5 marks each.

32. (a) Find the sub-interval of $\left(0, \frac{\pi}{2}\right)$ in which f(x) = log (sin x + cos x) is increasing and decreasing.

OR

(b) A rectangle of perimeter 30 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.



Sol. 32(a)

$f(x) = \log(\sin x + \cos x)$, $x \in \left(0, \frac{\pi}{2}\right)$

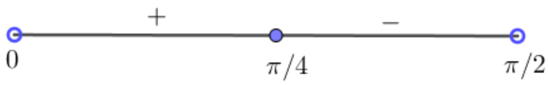
$\Rightarrow f'(x) = \frac{\cos x - \sin x}{\sin x + \cos x}$ — I — 2

For critical points of $f(x)$, put $f'(x) = 0$

$\Rightarrow \frac{\cos x - \sin x}{\sin x + \cos x} = 0 \Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$ — II — 1



$f'(x) > 0$ when $x \in \left(0, \frac{\pi}{4}\right) \Rightarrow f(x)$ is increasing in $\left(0, \frac{\pi}{4}\right)$. — III — 1

Note: Alternatively $f(x)$ is also increasing in the interval $\left(0, \frac{\pi}{4}\right]$.

$f'(x) < 0$ when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow f(x)$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. — IV — 1

Note: Alternatively $f(x)$ is also decreasing in the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$.

OR

Sol. 32(b)	Let the lengths of the sides of the rectangle be x and $(15-x)$. Let it be revolved around the side of length $(15-x)$, so that x becomes the radius of the cylinder. Volume of cylinder, $V=\pi x^2(15-x)=\pi(15x^2-x^3)$ $\frac{dV}{dx}=\pi(30x-3x^2)$ For critical points, put $\frac{dV}{dx}=0$ $\Rightarrow x=10\text{cm} \quad (\because x\neq 0)$ Now $\frac{d^2V}{dx^2}=\pi(30-6x)$; $\left.\frac{d^2V}{dx^2}\right]_{x=10\text{cm}}=-30\pi<0$ So, volume is maximum when $x=10\text{cm}$. Hence the dimensions of rectangle are 10 cm and 5 cm.	I II III IV V VI	1 1 ½ 1 1 ½
33.	Find the domain of q(x) = $\cos^{-1}(4x^2-3)$. Hence, find the value of x for which q(x) = 0. Also, write the range of 3q(x) – π.		
Sol.	Since domain of $\cos^{-1}x$ is $[-1,1]$. $\therefore -1\leq 4x^2-3\leq 1$ $\Rightarrow 2\leq 4x^2\leq 4$ $\Rightarrow \frac{1}{2}\leq x^2\leq 1$ $\Rightarrow -1\leq x\leq -\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}\leq x\leq 1$ $\therefore$ Domain of $q(x)=\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$ When $q(x)=0$, we have $\cos^{-1}(4x^2-3)=0$ $\Rightarrow 4x^2-3=\cos 0=1$ $\Rightarrow x^2=1\Rightarrow x=1,-1$ Since range of $q(x)=\cos^{-1}(4x^2-3)$ is $[0,\pi]$ i.e. $0\leq q(x)\leq\pi \Rightarrow 0\leq 3q(x)\leq 3\pi$ $\Rightarrow -\pi\leq 3q(x)-\pi\leq 2\pi$ Hence the required range is $[-\pi,2\pi]$.	I II III IV V	1 1 1 1 1

34.	A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.		
Sol.	The line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ is given by $l_1 : \frac{x-1}{5-1} = \frac{y-2}{8-2} = \frac{z-3}{11-3}$ *i.e.* $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \mu$	I	1
	Also the given line in its cartesian form is, $l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \lambda$ Any point on l_1 is $P(2\mu+1, 3\mu+2, 4\mu+3)$ Any point on l_2 is $Q(5\lambda+4, 2\lambda+1, \lambda)$	II	½
	When the lines intersect, the points P and Q must coincide. $\therefore 2\mu+1 = 5\lambda+4, 3\mu+2 = 2\lambda+1$ and $4\mu+3 = \lambda$ $\Rightarrow 2\mu - 5\lambda = 3 .....(\text{i})$, $3\mu - 2\lambda = -1 ......(\text{ii})$ and $4\mu - \lambda = -3 ......(\text{iii})$ solving any two above equations, we get $\mu = \lambda = -1$	III	1
	The point of intersection is $(-1, -1, -1)$.	IV	½
	Let the required line (l) passing through $(-1, -1, -1)$ be $\frac{x+1}{a} = \frac{y+1}{b} = \frac{z+1}{c}$ Since $l \perp l_1$ and $l \perp l_2$, $2a + 3b + 4c = 0, 5a + 2b + c = 0$ $\Rightarrow \frac{a}{3-8} = \frac{b}{20-2} = \frac{c}{4-15}$ *i.e.* $\frac{a}{-5} = \frac{b}{18} = \frac{c}{-11}$	V	1
	Hence, the required line is $\frac{x+1}{-5} = \frac{y+1}{18} = \frac{z+1}{-11}$ or $\frac{x+1}{5} = \frac{y+1}{-18} = \frac{z+1}{11}$ Note: The required line may also be written in the vector form. i.e. $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(5\hat{i} - 18\hat{j} + 11\hat{k})$ Note: Student can alternatively use point B to write the equation of line l_1.	VI	1

35.	(a) If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices : $x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7$ **OR** (b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.		
Sol. 35(a)	$QP = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ i.e. $QP = 6I$	I	2
	$\Rightarrow P^{-1} = \frac{1}{6}Q$	II	½
	Given system of equations is given by $PX = R$ where, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, R = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$	III	½
	$PX = R \Rightarrow X = P^{-1}R$ $\Rightarrow X = \frac{1}{6}Q.R$ $= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$	IV	½
	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x = 2, y = -1, z = 4$	V	1½

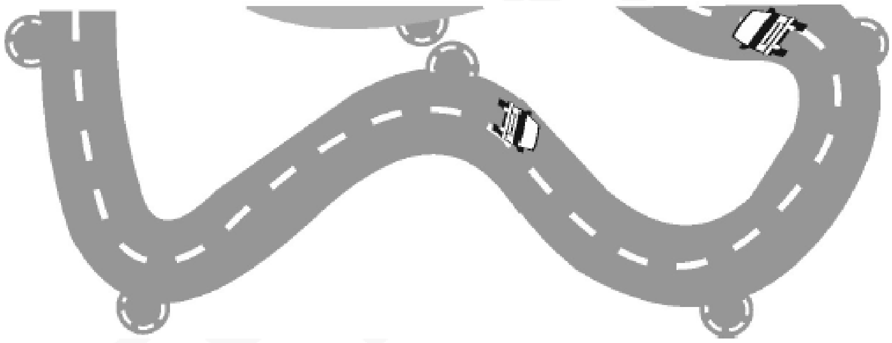
OR

Sol. 35(b)	$\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$		
	$=(1+x)\left[(1+y)(1+z)-1\right]-1\left[(1+z)-1\right]+1\left[1-(1+y)\right]$	I	2
	$=(1+x)(1+z+y+yz-1)-z-y$		
	$=xy+yz+zx+xyz$	II	1
	$\Delta=0 \Rightarrow xy+yz+zx+xyz=0$		
	$\Rightarrow yz+zx+xy=-xyz$	III	1
	Dividing both sides by xyz,		
	$x^{-1}+y^{-1}+z^{-1}=-1$	IV	1
	Note: Proportionate marks may be allotted if a student uses properties of determinants to solve this question.		

SECTION E

This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.

36. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.



The equation of one such track is given as follows :

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \le x < 3 \\ x^2 + 40 \quad , & x \ge 3 \end{cases}$$

Based on given information, answer the following questions :

(i) Find $f'(x)$ for $0 < x < 3$. **1**

(ii) Find $f'(4)$ **1**

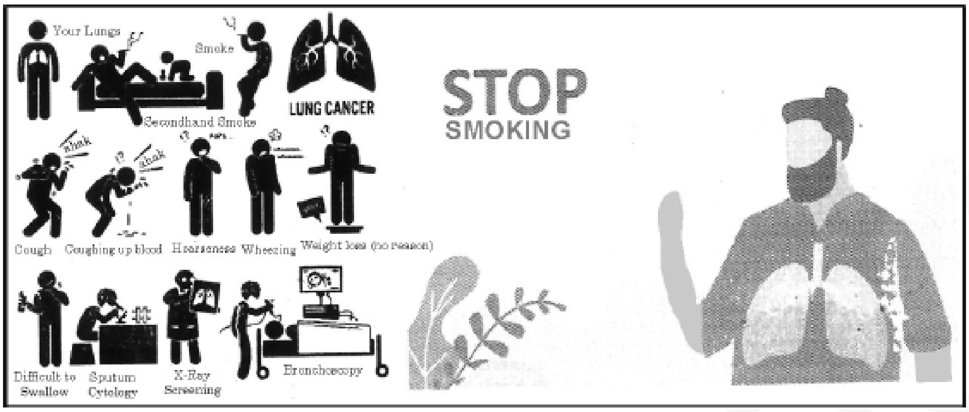
(iii) (a) Test for continuity of $f(x)$ at $x = 3$. **2**

OR

(iii) (b) Test for differentiability of $f(x)$ at $x = 3$. **2**

Sol.	(i) For $0<x<3, f(x)=x^4-4x^2+4$ $\therefore f'(x)=4x^3-8x$	I	1
	(ii) For $x\geq 3, f(x)=x^2+40$ $\therefore f'(x)=2x$ Hence, $f'(4)=2\times 4=8$	I	1
	$(iii)(a)$ Here $f(3)=49$ LHL: $\lim\limits_{x\to 3^-}(x^4-4x^2+4)=49$ RHL: $\lim\limits_{x\to 3^+}(x^2+40)=49$	I	1½
	Since $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x)=f(3)$, so $f(x)$ is continuous at $x=3$.	II	½
	OR		
	$(iii)(b)$ Here $f(3)=49$ getting LHD $=84$ getting RHD $=6$	I	1½
	Since LHD $\neq$ RHD at $x=3$, so $f(x)$ is not differentiable at $x=3$.	II	½

37.	Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.

A person is selected at random from these 50 people and tested for lung related problems.

Based on the given information, answer the following questions :

(i) What is the probability that selected person is a female ? **1**

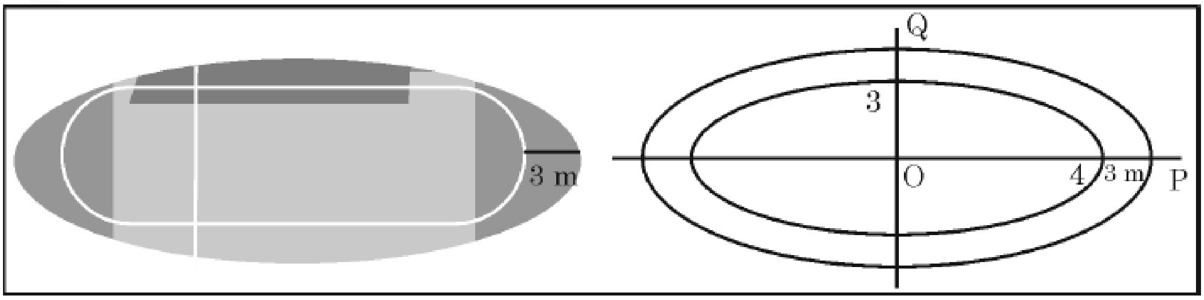
(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems ? **1**

(iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female. **2**

OR

(iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male. **2**

Sol.	Total number of smokers in the colony $=50$,		
	Number of male smokers $=30$, Number of female smokers $=20$		
	Let L represent the event that the person is suffering from the lung complication		
	$(i)P(F)=\dfrac{20}{50}=\dfrac{2}{5}$	I	1
	$(ii)P(L'\|M)=1-P(L\|M)=1-\dfrac{170}{1000}=\dfrac{83}{100}$	I	1
	$(iii)(a)\ P(L\|M)=\dfrac{170}{1000}=\dfrac{17}{100}$ and $P(L\|F)=\dfrac{120}{1000}=\dfrac{12}{100}$		

	Using Baye's Theorem, $\mathrm{P}(F\mid L)=\dfrac{P(F).P(L\mid F)}{P(M).P(L\mid M)+P(F).P(L\mid F)}$ $=\dfrac{\frac{2}{5}.\frac{12}{100}}{\frac{3}{5}.\frac{17}{100}+\frac{2}{5}.\frac{12}{100}}$ $=\dfrac{24}{75}$ or $\dfrac{8}{25}$ OR $(iii)(b)$ Here $P(L'\mid M)=\dfrac{83}{100}, P(L'\mid F)=\dfrac{88}{100}$ Using Baye's Theorem, $\mathrm{P}(M\mid L')=\dfrac{P(M).P(L'\mid M)}{P(M).P(L'\mid M)+P(F).P(L'\mid F)}$ $=\dfrac{\frac{3}{5}.\frac{83}{100}}{\frac{3}{5}.\frac{83}{100}+\frac{2}{5}.\frac{88}{100}}$ $=\dfrac{249}{425}$	I II I II	1½ ½ 1½ ½
38.	A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :  		

	Based on given information, answer the following questions : (i) Express y as a function of x from the given equation of ellipse. **1** (ii) Integrate the function obtained in (i) with respect to x. **1** (iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration. **2** **OR** (iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration. **2**		
Sol.	$(i)\,9x^2+16y^2=144 \Rightarrow y=\frac{3}{4}\sqrt{16-x^2}$	I	1
	$(ii)\int y\,dx=\int\frac{3}{4}\sqrt{16-x^2}\,dx$ $=\frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\left(\frac{x}{4}\right)\right]+C$	I	1
	$(iii)(a)$ Required Area $=4\times\int_0^4\frac{3}{4}\sqrt{16-x^2}\,dx$	I	1
	$=3\left[\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $=3\left(8\sin^{-1}1-0\right)=24\times\frac{\pi}{2}=12\pi$	II	1
	OR $(iii)(b)$ The track has a width of 3 m. Clearly $P(7,0)$ and $Q(0,6)$	I	1
	The equation of line passing through $P(7,0)$ and $Q(0,6)$ is $\frac{x}{7}+\frac{y}{6}=1$ Here, $y=\frac{6}{7}(7-x)$ Required area $=\int_0^7 y\,dx=\int_0^7\frac{6}{7}(7-x)\,dx$	II	½
	$=\frac{6}{7}\left[\frac{(7-x)^2}{-2}\right]_0^7$ $=21$	III	½